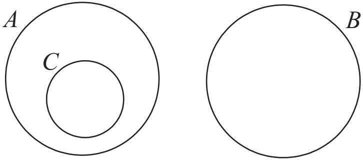
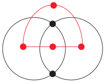
1.3 Introducing Euler Diagrams

An Euler diagram

This Euler diagram has three simple closed curves each of which has a unique label.

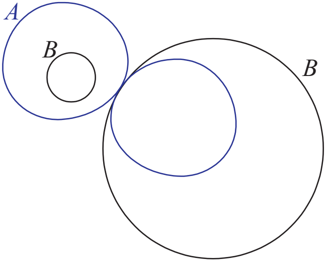


A closed curve is simple if it does not self-intersect.

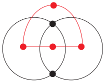


Another Euler diagram

This Euler diagram has three closed curves each of which has a label.

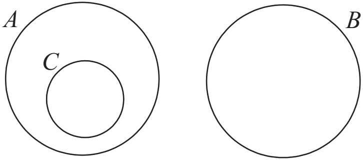


The curve labelled A is non-simple and the label B is used twice.



Definition: Euler diagram

An Euler diagram is a pair, $(Curve, l)$ where Curve is a set of closed curves (in $\mathbb{R}^2$) and l is a function that returns the label of each curve.



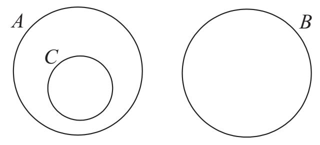
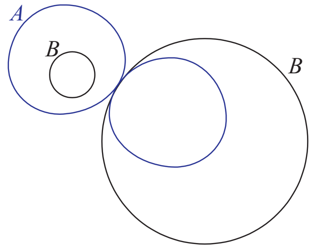
This Euler diagram has three curves, c_1, c_2 and c_3, where $l(c_1) = A$, $l(c_2) = B$ and $l(c_3) = C$.

Key concepts

- Contours
- Minimal Regions
- Zones
- Euler graph
- Dual graph

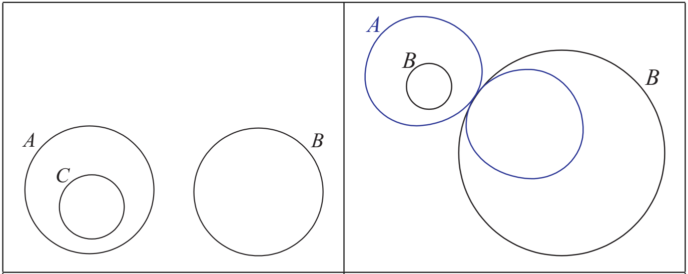
Key concept 1: Contours

A contour with label λ is a the set of curves in a diagram with label λ.

	
three curves three contours	three curves two contours

Key concept 2: Minimal Regions

A minimal region is a connected component of the plane formed by the (images of the) curves.

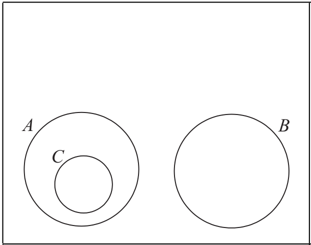


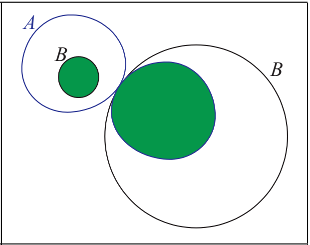
four minimal regions	five minimal regions

Each minimal region can be described as being inside certain curves (possibly none) and outside the rest of the curves.

Key concept 3: Zones

A zone is component of the plane that can be described as being inside certain contours (possible none) and outside the rest of the contours.





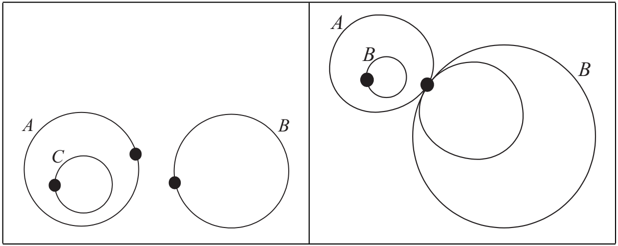
four minimal regions four zones	five minimal regions four zones

Each zone is a set of minimal regions.

Note: A point is inside a contour if the number of its curves it is inside is odd.

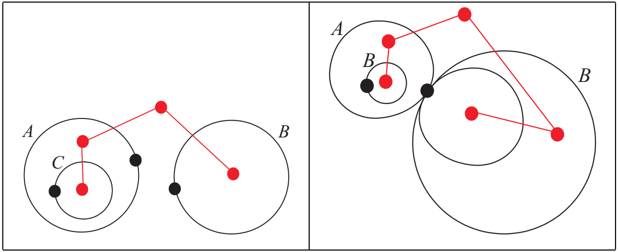
Key concept 4: Euler Graph

An Euler graph is a plane graph whose edges are formed by the curves of the Euler diagram.



Key concept 5: Dual Graph

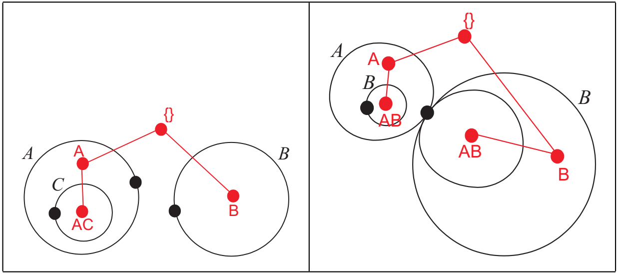
A dual graph is a dual of an Euler graph.



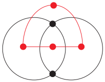
Note: We label the vertices and edges of the dual graph.

Key concept 5: Dual Graph

Labelling the vertices.

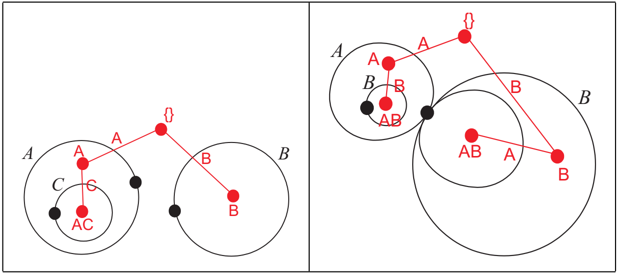


The vertices are labelled by the set of labels of the contours that they are inside.



Key concept 5: Dual Graph

Labelling the edges.

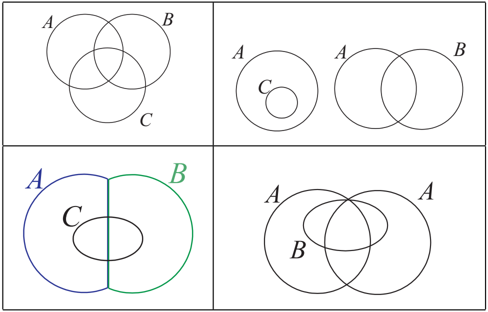


The edges are labelled by the symmetric difference of the vertex label sets.

Problems

For each of the Euler diagrams below,

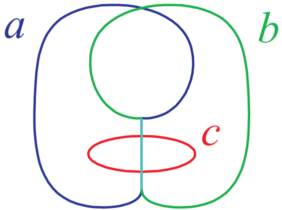
1. Write down how many contours the diagram possesses.
2. Shade the minimal region(s) that are inside curve(s) labelled A only.
3. Shade the zone that is inside contour A only.
4. Identify how many minimal regions and zones the diagram possesses.
5. Draw an Euler graph and dual graph, including labels on the vertices.

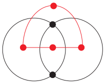


Properties of Euler Diagrams

No concurrency

No pair of curves run concurrently.

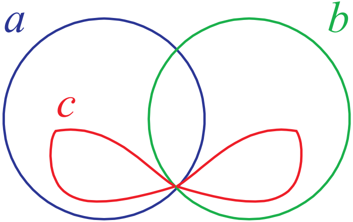




Properties of Euler Diagrams

Simplicity

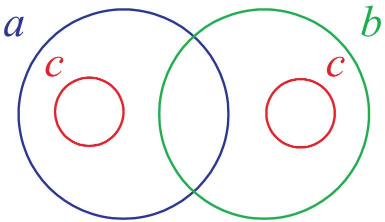
All of the curves are simple.

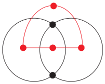


Properties of Euler Diagrams

Unique labels

Curve labels are unique.

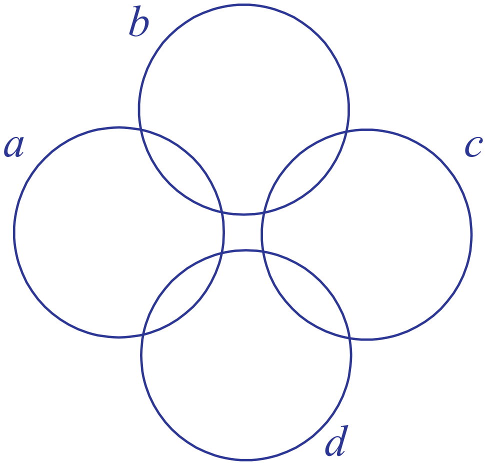




Properties of Euler Diagrams

Connected zones

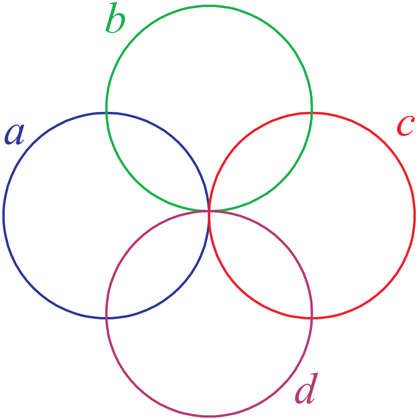
Each zone is connected

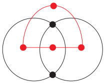


Properties of Euler Diagrams

No triple-points

There are no triple-points of intersection between the curves.

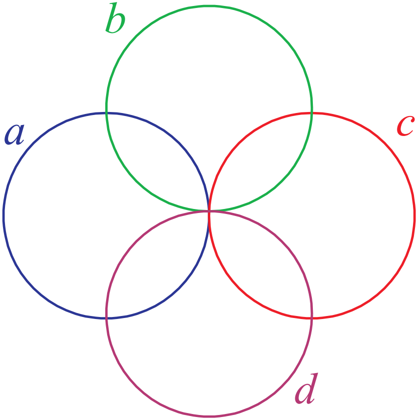




Properties of Euler Diagrams

No brushing points

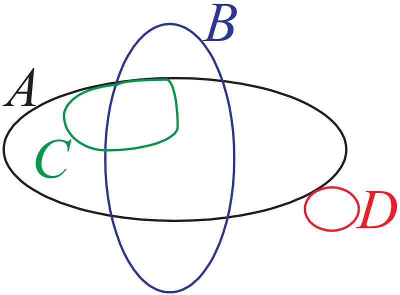
Whenever two curves meet at a point, they cross.

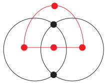


Properties of Euler Diagrams

Possessed Simplicity, Unique labels

Not Possessed No concurrency, Connected zones, No triple points, No brushing points

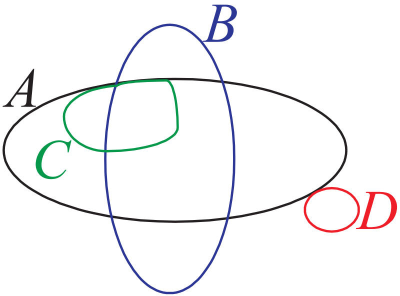




Counting Violations

Properties of Euler Diagrams

Property	Count
No concurrency	2
Simplicity	0
Unique labels	0
Connected zones	2
No triple points	1
No brushing points	1



Problems

For each of the Euler diagrams below, identify which of the six properties are possessed and count the number of times each of the other properties are violated.